Appendix

Applicant has referred to the book by R. L. Mills entitled *The Grand Unified Theory of Classical Quantum Mechanics* (Blacklight Power Inc., New Jersey, 1999; hereafter, "GUT") which describes the existence of new energy states for the hydrogen atom that are below the conventionally accepted ground state energy. A hydrogen atom in any one of these new energy states is termed a "hydrino." According to equations (I.75a-c) on pages 19-20 of GUT, the general formula representing the energy levels for an electron with a principal quantum number, n, around the nucleus of the hydrogen atom is:

$$E_n = -\text{(Rydberg constant)}/n^2 = -13.6 \text{ electron volt}/n^2$$

where n = 1, 2, 3, etc and n is also = 1/2, 1/3, 1/4, etc. While the former integer values of n give energies that are conventionally understood and experimentally verified, the latter fractional values of n lead to the energies of the electron in a hydrino atom which, according to Mills, represents a new "lower energy hydrogen atom."

A review of some of the main mathematical underpinnings in GUT shows that there is really no proper theoretical basis to assert the existence of the hydrino atom in view of the following discussion.

Nowhere has Mills satisfactorily established that fractional values of n arise as a natural consequence of a logical and internally consistent mathematical and scientific framework. While GUT bristles with a dense array of mathematical equations, the fractional values of n are not shown to be the unequivocal end result of Mills' theory. It appears that there is an internal break in logic in the mathematical analysis, with Mills ultimately relying on conclusionary statements, such as, a nonradiative boundary condition and the relationship between the electron and a photon gives transitions in which the electron goes to a "lower" energy nonradiative state with a smaller radius or, alternatively, that an electron can undergo a collision with an "energy hole" which allows the electron to undergo a transition to a lower energy nonradiative state with a smaller radius (pages 16-17 of GUT). In these transitions,

the process involved is called a "shrinkage reaction" yielding a shrunken hydrogen atom accompanied by the release of energy. See pages 16, 17 and 144-146 of GUT.

By way of background, it is noted that there are at least two conventionally recognized approaches to the problem of obtaining the energy levels of the electron in the hydrogen atom. These are:

(a) *Via* a Differential Equation approach formulated as a two-point boundary value problem where boundary conditions at the nucleus and at infinity are imposed on the radial wavefunction of the electron which satisfies a second-order linear differential equation known as Schrödinger's wave equation. It is to be understood that while the complete wavefunction in spherical polar coordinates is the product of a radial wavefunction and angular wavefunctions, the complete wavefunction for the ground (or lowest energy) state of the hydrogen atom is independent of angular coordinates in view of the spherical symmetry of that state, and is studied only on the basis of the radial wavefunction. Thus, see attached sections 18d-18e and 21b at pages 121-124 and 139 from Pauling and Wilson's *Introduction to Quantum Mechanics* (Dover Publications, Inc., New York, 1985) and **Endnote 1**.

(b) *Via* an Integral Equation approach wherein the boundary conditions on the radial wavefunction of the electron are "built into" the integral equation itself rather than being imposed on it as in the differential equation formulation. In this approach, upon taking the Fourier transform of the wavefunction, subject to the boundary condition that it satisfies Schrödinger's equation, an integral equation is obtained. Thus, see attached pages 899-900 from Morse and Feshbach's *Methods of Theoretical Physics, Part I* (McGraw-Hill Book Company, New York, 1953) and **Endnote 2**.

It is crucial to note that either approach is but a mathematical tool and that, while the integral equation approach may be mathematically more compact and, perhaps, be more convenient for solving certain problems compared to the differential equation approach, the

final results given by either approach must not be mutually contradictory if a scientific theory based on these approaches is to be logical and internally consistent.

From a consideration of Mills' mathematical derivations on pages 4-5 (equations (I.5) to (I.11)) , on pages 32-38 (equations (1.3) to (1.45)) and on pages 136-141 (equations (5.1) to (5.21)) of GUT, it appears that Mills' formulation *may* be an integral equation type of approach. Specifically, the boundary condition "built into" the integral equation is an expression for the current density, and, thus, the charge density of a point charge which satisfies Maxwell's equation for the electric field as given by Haus in a paper, in the *American Journal of Physics*, vol. 54, no. 12, pages 1126-1129 (1986), relating to the absence of radiation from a point charge moving at constant velocity. See page 3 of GUT. While, Haus' paper is not the focus of discussion here, it is apparent that the use of a Dirac delta function, $\delta(r-r_n)$, to represent the electron charge density on page 4 of GUT may be an unphysical assumption in that, whereas the electron charge density is an "observable" that is ultimately measurable, the delta function, which purports to represent it, is not, in and of itself, a function in the usual mathematical sense of the term and is physically meaningful only under an integral sign.

More specifically, it appears that Mills' integral equation approach utilizes the technique of the "Green's function." In the theory of integral equations, the Green's function is a function that satisfies a differential equation involving a Dirac delta function type of point source. A connection between the Green's function and the wavefunction is established by requiring the former to satisfy boundary conditions corresponding to those satisfied by the latter. Interpreting Mills' equations as best as one can, it is possible, though by no means certain, that Mills achieves such a connection by requiring the Green's function to satisfy boundary conditions imposed on the charge density function in Mills' equation (1.1) on page 31 of GUT. The final step in the integral equation approach is to generate an integral

equation involving an integral taken over the Green's function. The solution of that equation would yield the wavefunction of the electron and, from that, leads to the energy levels of the electron in the hydrogen atom. See attached pages 808, 902 and 903 from Morse and Feshbach *op. cit.* and **Endnote 3**. It is observed that the legitimate use of a Green's function which satisfies an equation involving a Dirac delta function type of "point source" and appears, ultimately under an integral sign as the kernel of an integral equation, does not justify Mills' representation of the electron charge density, which is a "smeared out" charge distribution, as a Dirac delta function as discussed previously. Mills' lack of consistency in using properly subscripted variables as well as the absence of a logical flow in the mathematical derivations, prevents one from properly assessing the kind of approach taken in GUT.

In any event, at least some problematical issues are seen in the Mills' treatment, *viz.*, (i) it is not explained as to why it is physically meaningful to utilize Haus' boundary condition for a classical point charge moving in free space in order to obtain the energy levels of the electron in a quantized system such as the hydrogen atom where the electron moves in a confined space due to its attractive coulombic interaction with the positively charged nucleus, and, (ii) there is no explanation for the catastrophic collapse of the electron into the nucleus as $n \rightarrow \infty$ in the fractional quantum number series, $1/n$, i.e. the hydrino atom implodes and ceases to exist. See pages 144-146 of GUT. The end result of Mills' integral equation approach, if such it is, fails to bear out his assertion that n must unequivocally have fractional values. In essence, it appears that the condition that n have fractional values (see equations (1.75c) and (2.2) on pages 20 and 81 of GUT) is but an *ad hoc* statement that does not logically flow from Mills' derivation of the equation for the energy levels of the electron in the hydrogen atom and it may even represent a type of forced parameterization scheme

deliberately structured to produce a desired outcome contrary to the logical flow of its mathematics or, even, common sense.

Hence, it appears that Mills' theory remains essentially unproven as discussed above and does not constitute a proper basis to demonstrate the existence, at least on theoretical grounds alone, of the so-called hydrino atom.

Furthermore, Mills' theory does not show that the conventional quantum mechanical treatment of the hydrogen atom is theoretically or experimentally flawed. Any attempt to establish a new result for the hydrogen atom that is presently unknown to quantum mechanics must cross a rather steep threshold of scientific credibility. See the attached page 2 from Bethe and Salpeter's *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977 and **Endnote 4**.

Among the many problems solved by quantum mechanics, the hydrogen atom, along with the linear harmonic oscillator and the particle-in-a-box, is one of the few scientific problems that has received extensive theoretical and experimental treatment over many years since the first decade of the twentieth century. For a complete treatment of the hydrogen atom problem see the attachment from pages 19-1 to 19-18 of Feynman's *Lectures in Physics, vol. III, Quantum Mechanics* (Addison-Wesley Publishing Co., Reading, Mass., 1965). The results obtained from at least one type of standard procedure for solving the radial Schrödinger equation using a power series expansion for the wavefunction of the electron inescapably lead to the conclusion that only positive integer values for n are permissible (as explained previously in **Endnote 1)**. See attached pages 1-9 and 2-6 from Feynman *op. cit.* and **Endnote5**. In other words, conventional theory and experiment forbid hydrino atoms.

Endnote 1

Schrödinger's wave equation for the radial wavefunction, $S(\varrho)$, is:

$$(1/\varrho^2)(d/d\varrho)(\varrho^2 dS/d\varrho) + \{-1/4 - l(l+1)/\varrho^2 + \lambda/\varrho\}S = 0$$

where ϱ is proportional to the radial coordinate in the spherical polar coordinate system with $0 \leq \varrho \leq \infty$, l is the orbital angular momentum quantum number and λ is proportional to negative (i.e. bound) energy values. The boundary conditions are that far from the nucleus of the hydrogen atom ($\varrho \to \infty$) the radial wavefunction becomes negligible i.e. $S \to 0$, and, at the nucleus of the atom ($\varrho = 0$), noting that S is expressible as $e^{-\varrho/2}\varrho^s L(\varrho)$ where $L = \sum_{v} a_v \varrho^v$ is an infinite power series in ϱ, substitution of the expression for S into the radial wavefunction equation results in the choice of $s = +l$ (which is a positive integer) as the only choice that will permit to be S be an acceptable wavefunction, which in turn yields the boundary condition that S has a finite value at the nucleus. Note that despite the finite value of the radial wavefunction at the nucleus, the probability of finding the electron at the nucleus, $\varrho = 0$, of the hydrogen atom in its normal ground state is proportional to $4\pi\varrho^2 S^2$ which, of course, is zero. Upon substituting the cited expression for S into the radial wavefunction equation, recursion relations between a_v for various values of v are obtained. The recursion relations contain the principal quantum number n appearing as a multiplicative coefficient of a_v. Since S must have a proper asymptotic behavior as $\varrho \to \infty$, this requires that the infinite power series be terminated after a finite number of terms which in turn, after some algebra, leads to the result that n must be a positive integer having the values 1, 2, 3, etc.. See equations (18.29) to (18.39) and Figure 21-1 at pages 121-124 and 140 in Pauling and Wilson.

Endnote 2

Substitution of the Fourier transform of the wavefunction, $\psi(\mathbf{r})$, viz.

$$\psi(\mathbf{r}) = (1/h)^{3/2} \int_{-\infty}^{\infty} \varphi(\mathbf{p}) e^{(2\pi i/h)\mathbf{p} \cdot \mathbf{r}} d\mathbf{p},$$

where h is Planck's constant and $\mathbf{p}$ and $\mathbf{r}$ are momentum and spatial coordinate vectors, respectively, into the Schrödinger equation in the differential form

$$\nabla^2\psi + (2m/h^2)\{E - V[\mathbf{r}, (h/2\pi i)\nabla]\}\psi = 0,$$

where ∇^2, E and V are the Laplacian operator, total and potential energies, respectively, followed by multiplication through by $(1/h)^{3/2} e^{(-2\pi i/h)\mathbf{q} \cdot \mathbf{r}}$ and an integration over $\mathbf{r}$ yields the desired integral equation

$$(q^2/2m)\varphi(\mathbf{q}) + \int_{-\infty}^{\infty} \varphi(\mathbf{p})V(\mathbf{p}\text{-}\mathbf{q}, \mathbf{p})d\mathbf{p} = E\varphi(\mathbf{q})$$

where

$$V(\mathbf{p}\text{-}\mathbf{q}, \mathbf{p}) = (1/h)^{3/2} \int_{-\infty}^{\infty} e^{(2\pi i/h)(\mathbf{p}\text{-}\mathbf{q}) \cdot \mathbf{r}} V(\mathbf{r},\mathbf{p})d\mathbf{r}$$

with $\mathbf{q}$ being a momentum vector.

See equation (8.1.4) at page 900 in Morse and Feshbach.

Endnote 3

To illustrate a method of obtaining a solution for the wavefunction, ψ, by the technique of Green's functions consider the Schrödinger equation for ψ written as:

$$[\nabla^2 + k^2]\psi = U\psi$$

where $k^2 = (8\pi^2 m/h^2)E$ and $U = (8\pi^2 m/h^2)V$ with E and U being the total and potential energies, respectively. A Green's function, $G_k(\mathbf{r}|\mathbf{r}_0)$, is introduced which satisfies

$$[\nabla^2 + k^2]G_k(\mathbf{r}|\mathbf{r}_0) = -4\pi\delta(\mathbf{r}\text{-}\mathbf{r}_0),$$

where $\delta(\mathbf{r}\text{-}\mathbf{r}_0)$ is a Dirac delta function representing a "point source" at $\mathbf{r}_0$. The Green's function can be thought of as representing an effect at $\mathbf{r}$ caused by a point source at $\mathbf{r}_0$. The

boundary conditions on $G_k(r \mid r_0)$ are chosen to be the same as those corresponding to the boundary conditions on the wavefunction ψ. Then, by the theory of integral equations, a solution to the Schrödinger equation is:

$$\psi(r) = -(1/4\pi) \int G_k(r \mid r_0) U(r_0) \, \psi(r_0) dr_0.$$

See pages 808, 902 and 903 in Morse and Feshbach.

Endnote 4

Regarding the study of the hydrogen atom, note the following quotation from page 2 of Bethe and Salpeter's classic text entitled *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977):

"One of the simplest, and most completely treated, fields of application of quantum mechanics is the theory of atoms with one or two electrons. For hydrogen and the analogous ions He^+, Li^{++}, etc., the calculations can be performed exactly, both in Schrödinger's nonrelativistic wave mechanics and in Dirac's relativistic theory of the electron. More specifically, the calculations are exact for a single electron in a fixed Coulomb potential. Hydrogen-like atoms thus furnish an excellent way of testing the validity of quantum mechanics. For such atoms the correction terms due to the motion and structure of atomic nuclei and due to quantum electrodynamic effects are small and can be calculated with high accuracy. Since the energy levels of hydrogen and similar atoms can be investigated experimentally to an astounding degree of accuracy, some accurate tests of the validity of quantum electrodynamics are also possible."

Endnote 5

It is noteworthy that this position is also supported by a different line of argument that is independent of the solution to Schrödinger's equation. Thus, fractional values for the principal quantum number n would bring the electron much closer to the nucleus of the hydrogen atom than is permitted by Heisenberg's Uncertainty Principle. Feynman has presented a mathematically simple argument, in his "Lectures in Physics," vol. III, page 2-6, to show that the size of the hydrogen atom i.e. when n is 1 (rather than, say, 1/2) is perfectly consistent with the Uncertainty Principle. This argument goes as follows: from the Uncertainty Principle, if the electron is at a distance a from the hydrogen nucleus, then the product of its momentum and a must be of the order of Planck's constant. Now the total energy of the electron is the sum of its kinetic and potential energies. Noting that the kinetic energy can be expressed in terms of the square of the momentum, upon invoking the value of the momentum from the Uncertainty Principle and minimizing the total energy in order to obtain the lowest energy level of the electron, one immediately obtains the standard result for the lowest energy level of the electron in the hydrogen atom which is consistent with n being 1 and no lower than 1. Since, according to Feynman, "no one has ever found (or even thought of) a way around the Uncertainty Principle ... so we must assume it describes a basic characteristic of nature," (page 1-9 in Feynman) it appears that Mills' fractional value for n is impermissible in light of the inviolability of the Uncertainty Principle.

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